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Structural Optimization of Guyed Trusses

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This paper describes the nonlinear formulation and sensitivity analysis for the integrated total Lagrangian analysis and design of geometrically nonlinear cable supported trusses such as transmission lines or antennas. For the purpose of automating the design process, the sequential quadratic programming method by Arora as well as a recently developed algorithm, the successive approximation method by Snyman and Stander, are used. Two simple illustrative truss examples are used to show how the use of a nonlinear as opposed to a linear formulation can result in an improved design methodology. A third example illustrates the use of Green–Lagrange prestrain as a design variable. Two further more complex examples are included to demonstrate the optimal sizing and pretensioning of cable-stayed trusses. Multiple load cases are considered.

Introduction

THERE is a strong trend in the design of skeletal structures to, wherever practical, make increasing use of tensioned cables. One of the best examples of the convergence of structural efficiency to a best design can probably be found when studying the evolution of the configurations of power transmission towers. From the early days of cantilever truss structures these have now evolved to the much lighter "cross rope" type in which two separate vertical cable-stayed lattice struts support the power conductors through suspension from a cross rope. Apart from the savings in material, significant savings in transportation cost can also be obtained especially since erection often has to occur in remote areas.

Although the cable-stayed structures are easy to analyze, the determination of a best or least weight design can be complex due to the variety of design variables available. These are, typically, configuration, geometry, member sizing, and prestraining. The optimization of a configuration is complex and is best accomplished manually, perhaps with the aid of layout optimization methods. On the contrary, geometric and prestrain optimization and sizing can be treated by direct sensitivity analysis of a chosen topology. This paper describes the sizing and prestrain optimization of guyed trusses. For the nonlinear analysis the total Lagrangian method is used in combination with the pure Newton method and adaptive arc-length procedures. Because of the lack of stiffness of a cable structure in its undeformed state, soft spring elements are used to stabilize the structure temporarily during the load stepping phase.

The choice of prestrain instead of prestress variables is justified by both the simplicity of inclusion of prestrain in the sensitivity analysis and by the independence of the prestrain quantity with respect to the sequence of pretensioning of the structure. The prestraining of a member can also simply be seen as the lack of fit of a member in a fixed geometry and can, in practice, be accomplished by specifying member lengths to create a predetermined lack of fit or by specifying turns of a turnbuckle included in the member.

For the optimization process, two optimization methods are used, namely, the sequential quadratic programming (SQP) method IDESIGN (PLBA) authored by Arora[1] and the successive approximation method (SAM) algorithm of Snyman and Stander[2] and Stander et al.[3] Using two methods gives optima in which greater confidence can be placed. Additionally, some idea of the degree of difficulty of the newly formulated optimization problems is gained.

Five examples are given to demonstrate the analysis and optimization methodology. The optimization of example structures subjected to second-order effects show that unexpected results can be obtained.

A number of papers,[4–8] related to the general topic of the sensitivity analysis and optimization of structures with nonlinear geometric response, have appeared in the literature. None of this previous work, however, is specific to cable-stayed trusses.

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Variational Formulation and Finite Element Analysis

Consider a continuous structure with external loads R^E and a load factor λ applied at discrete points and with an internal stress state represented by the tensorial components of the contravariant second Piola–Kirchhoff stress $S^{\alpha\beta}$. For a variation in geometry $\delta(\Delta x_S)$ over the volume V the variational equation can be written as

$$\int_V \left[S^{\alpha\beta}\delta\epsilon_{\alpha\beta} - \lambda R_S^E \delta(\Delta x_S)\right] \mathrm{d}V = 0; \qquad S = 1, 2, \ldots, n \quad (1)$$

where $\delta\epsilon$ represents the covariant Green strain variations compatible to $\delta(\Delta \boldsymbol{x})$. The summation convention applies to all repeated indices.

As a result of the linearization of Eq. (1) and substitution of the constitutive and material relationships

$$S^{\alpha\beta} = C^{\alpha\beta\gamma\delta}\left(\epsilon_{\gamma\delta} - \lambda\epsilon_{\gamma\delta}^0\right) \quad (2)$$

and

$$2\epsilon_{\alpha\beta} = j_{\alpha i} j_{\beta i} - J_{\alpha i} J_{\beta i} \quad (3)$$

the equilibrium equation is derived in component form as

$$\int_V j_{\alpha i,S}\, j_{\beta i} S^{\alpha\beta}\, \mathrm{d}V + \int_V \left(j_{\alpha i,S}\, j_{\beta i} C^{\alpha\beta\gamma\delta} j_{\gamma j} j_{\delta j,T} + S^{\alpha\beta} j_{\alpha i,S}\, j_{\beta i,T}\right) \mathrm{d}V\, \Delta x_T = \lambda R_S^E$$
$$\alpha, \beta, \gamma, \delta = 1, 2, 3; \qquad S, T = 1, 2, \ldots, n \quad (4)$$

$C^{\alpha\beta\gamma\delta}$ is the constitutive tensor and $J_{\alpha i}$ and $j_{\alpha i}$ represent the Jacobian matrices of the undeformed and deformed states, respectively, namely,

$$J_{\alpha i} = \frac{\partial X_i}{\partial \xi_\alpha}; \qquad j_{\alpha i} = \frac{\partial x_i}{\partial \xi_\alpha}, \qquad \alpha = 1, 2, 3 \quad (5)$$

The total number of degrees of freedom of the model is n and the subscripts S or T refer to nodal changes in geometry x_S or x_T of the discretized model. Therefore, $j_{\alpha i,S}$ denotes the Jacobian operator $\partial j_{\alpha i}/\partial x_S$. Here ξ_α denote the curvilinear coordinates which, for convenience, are finite element based.

The external load factor λ is applied to all loads including the prestrains so that continuation of the response can be exercised over all components of the loading of the structure. When written in matrix form, Eq. (4) becomes

$$\boldsymbol{K}_T^{(i)} \Delta \boldsymbol{x}^{(i)} = \lambda(\boldsymbol{P} + \boldsymbol{D} + \boldsymbol{Q}^{(i)}) - \boldsymbol{R}^{I(i)} \quad (6)$$

where $K_T = K_E + K_G - \lambda K_G^0$ is the tangent stiffness and K_E and K_G are the elastic and geometric stiffness matrices, respectively. K_G^0 is the consistent prestrain stiffness matrix. The vector P represents the external "live" load and D the own weight, whereas Q is the equivalent prestrain load and R^I the internal force vector. In the iterative scheme the geometry is updated as

$$x^{(i+1)} = x^{(i)} + \Delta x^{(i)} \tag{7}$$

where $(i), (i+1), \ldots$ refer to consecutive iterations.

For cable and truss elements there is only one curvilinear dimension and, therefore, the formulation can be simplified by specifying $\alpha = \beta = \gamma = \delta = 1$, $S = S^{11}$ and $\epsilon = \epsilon_{11}$. For the isoparametric formulation, the Jacobian vectors for the current and undeformed states are

$$j = \phi_{,1}^k \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^k \qquad J = \phi_{,1}^k \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}^k \qquad k = 1, 2, \ldots, N \tag{8}$$

summing over k, where N is the number of nodes of an element and, ξ is the normalized local coordinate system of an element, ϕ^k represents the shape function $\phi^k(\xi)$ of node k and $\phi_{,1}^k = \partial\phi^k/\partial\xi$. The norms of the Jacobian vectors are given by $t = \|j\|$ and $T = \|J\|$.

The elastic stiffness submatrix (for nodes k and l) is

$$K_E^{kl} = \int_{-1}^{+1} \phi_{,1}^k \phi_{,1}^l \frac{AE}{T^3} j \otimes j \, d\xi \tag{9}$$

where $\otimes$ denotes the outer product, A is the cross-sectional area of the undeformed structure, and E is Young's modulus. Note that K_E is only of rank N but is potentially stabilized by adding the geometric stiffness of which a nodal submatrix (for nodes k and l) is defined as

$$K_G^{kl} = \int_{-1}^{+1} \phi_{,1}^k \phi_{,1}^l \epsilon \frac{AE}{T^3} I \, d\xi \tag{10}$$

where

$$2\epsilon = j \cdot j - J \cdot J = t^2 - T^2 \tag{11}$$

and I is a 3×3 unit matrix.

The expression

$$K_G^{0kl} = \int_{-1}^{+1} \phi_{,1}^k \phi_{,1}^l p \frac{AE}{T} I \, d\xi \tag{12}$$

defines the prestrain stiffness matrix where p is the specified Cartesian Green prestrain value. The internal force vector (for node k) is given by

$$R^{Ik} = \int_{-1}^{+1} \phi_{,1}^k \epsilon \frac{AE}{T^3} j \, d\xi \tag{13}$$

whereas the equivalent prestrain loading for a node k is given as

$$Q^k = \int_{-1}^{+1} \phi_{,1}^k p \frac{AE}{T} j \, d\xi \tag{14}$$

For the stress constraint formulation, the longitudinal Cauchy stress

$$\sigma = TtS \tag{15}$$

is required.

The equilibrium state x^* for a specific load factor can now be solved incrementally from Eqs. (6) and (7) by using pure or modified Newton methods.

Design Sensitivity Formulation

In terms of the sizing cross-sectional variables a and the prestrain variables p, Eq. (6) can be written as

$$\left[K_E(a) + K_G(a) - K_G^0(a,p)\right]\Delta x = P + D(a) + Q(a,p) - R^I(a) \tag{16}$$

since the load factor $\lambda = 1$ during optimization.

The equilibrium relationship as expressed by Eq. (16) can be written in terms of the design variables $b^T = [a^T, p^T]$ as

$$R[b, x(b)] = 0 \tag{17}$$

where x represents the deformed state and R represents the force imbalance. It can be seen that R is also an implicit function of the design variables. As was also shown in Ref. 5, total derivatives of Eq. (17) yield

$$\frac{dR}{db} = \frac{\partial R}{\partial b} + \frac{\partial R}{\partial x}\frac{dx}{db} = 0 \tag{18}$$

or

$$\bar{R} + K_T \bar{x} = 0 \tag{19}$$

The overbar symbol indicates derivatives (sensitivities) with respect to the design variables. The displacement sensitivity $\bar{x}$ can be determined from the internal and external force sensitivities by solving Eq. (19). The current tangent stiffness matrix K_T is known from the analysis and is determined at an equilibrium state from the assembly of Eqs. (9), (10), and (12). For sizing, the internal element force sensitivity vector is (for node k)

$$(Q - R^I)_{,a}^k = \int_{-1}^{+1} \phi_{,1}^k STj \, d\xi \tag{20}$$

where $S = (\epsilon - T^2 p)E/T^4$. For prestrain sensitivity, this vector becomes

$$Q_{,p}^k = \int_{-1}^{+1} \phi_{,1}^k \frac{AE}{T} j \, d\xi \tag{21}$$

where the $,p$ and $,a$ symbols denote $\partial()/\partial p$ and $\partial()/\partial a$, respectively.

The vectors of Eqs. (20) and (21) are assembled into a structural vector, and the displacement sensitivity is obtained from Eq. (19).

An important component of the stress sensitivity, which is required for constraint gradients, is the Green strain sensitivity. The natural Green strain ϵ, is given by Eq. (11) so that after discretization of the geometry, the strain sensitivity can be derived with some manipulation as

$$\frac{d\epsilon}{db} = j_l \phi_{,1}^k \bar{x}_l^k; \qquad k = 1, 2, \ldots, N, \qquad l = 1, 2, 3 \tag{22}$$

Using Eq. (15) the Cauchy stress sensitivities can then be calculated from the Green strain sensitivities as

$$\sigma_{,a} = \frac{E\epsilon_{,a}}{2T^3 t}[3t^2 - T^2(1 + 2p)] \tag{23}$$

and

$$\sigma_{,p} = \frac{E\epsilon_{,p}}{2T^3 t}[3t^2 - T^2(1 + 2p)] - E\frac{t}{T} \tag{24}$$

for sizing area and prestrain variables, respectively.

Computational Aspects

Design Iteration Strategy

The design procedure consists of a loading phase and an iterative optimization phase.

In the first phase the trial structure is loaded to almost its full prescribed load. In this phase, several steps of an adaptive arc-length control method[9] are used to firstly obtain a loading exceeding 80% of the full load whereupon the algorithm switches to a single load control step to obtain the equilibrium structural configuration at the exact load. During the arc-length stepping phase, all the degrees of freedom of the structure are held by "soft" artificial springs which are released before the final correcting load step of this phase. This precaution ensures stability of structures, such as cables, during the loading phase. Consistency of the tangent stiffness matrix and arc-length procedure is illustrated by the quadratic convergence of the Newton method in the region of equilibrium.

With the structure in equilibrium, a design change is calculated by a single step of the optimization algorithm. The resulting design is not in equilibrium and to recover equilibrium the current geometrical state and new design are used as inputs to a new nonlinear analysis using load control (at the full load) only. However, as the design change may be large enough to move beyond the region of convergence, a line search procedure[10] has been implemented as part of the Newton method to ensure global convergence. This is the beginning of the second phase, and from here on the cycle of optimization-step/equilibrium iteration is repeated at constant loading until convergence to the optimal design is achieved. In the implementation of the formulation, the pure Newton method was used in the equilibrium iteration procedure whereas both the SQP method IDESIGN[1] and successive approximation method[2,3] were used for optimization. Although normalized constraint equations are used in the sensitivity analysis, the design variables have not been scaled.

Multiple Load Cases

The implementation includes the handling of multiple load cases. These are simply treated by iterating to the equilibrium position for each case individually before constructing the constraint equations for that loadcase.

Example Problems

Five example problems are given to illustrate various aspects of the optimization of nonlinear structures. The optimization results as well as algorithm performance are given for each example. For the optimization algorithm SAM, a convergence tolerance of 10^{-6} is utilized for the error in the design vector. Similarly, the tolerance for the convergence parameter in SQP is also 10^{-6}. In the tables, NCE refers to the number of constraint set evaluations. In some instances the value for SAM is not given as SAM has not been implemented in the prestrain analysis code. All prestrain values are given in millistrain.

With regard to the choice of the order of element interpolation functions: for cables linear, quadratic, or cubic elements may be used, whereas linear ($N = 2$) elements are used as truss bars. Although the given formulation is general, linear elements were also used to model cables in the example problems.

The first two examples have typical truss configurations and are used to illustrate the influence of nonlinear behavior on truss sizing design for minimum weight. A third example is based on a standard benchmark problem in which prestrain is introduced whereas a further two examples are used to illustrate the prestrain and sizing optimization of cable-stayed structures.

Example 1: Truss

The first example is of a tall, narrow braced truss with three vertical bays and with a large vertical load and some horizontal loading (e.g., due to wind), see Fig. 1. This example illustrates the influence on design of the so-called second-order or P–δ effect in which the loading interacts with the deformed geometry. Buckling formulas are employed for the compression modeling of members (see the Appendix).[11] All vertical members belong to design group 1, the horizontal members belong to group 2, and the bracing belongs to group 3; $E = 200$ GPa, $\gamma = 78.50$ kN/m^3, and the stress limit is 200 MPa. Own weight is included. The minimum allowable cross-sectional area is 1 mm^2. In the figure the units are millimeters and Newton.

Table 1 Sizes and stress criticalities of example 1

Group	Linear Area	Linear Criticality	Nonlinear Area	Nonlinear Criticality
1	586.6	1.000	590.1	1.00
2	50.33	1.000	51.89	1.00
3	2.236	1.000	5.870	$\boxed{0.41}$
Weight	1413.98		1427.84	
NCE (SQP)	11		23	
NCE (SAM)	13		19	

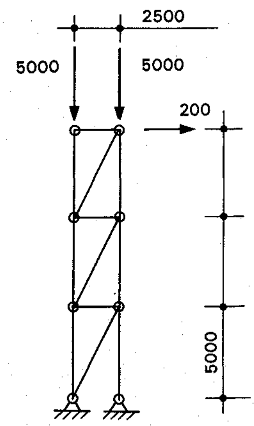

Fig. 1 Example 1: second-order effect.

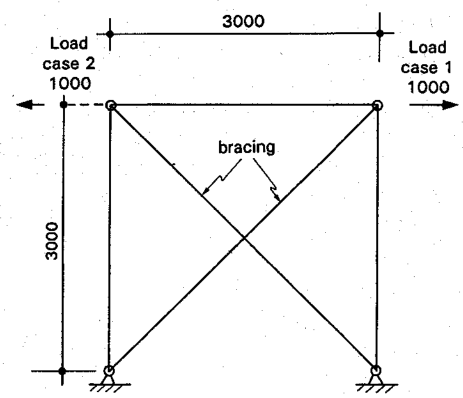

Fig. 2 Example 2: load shedding.

Table 1 shows the member sizes and stress criticalities for the two models used for the first example. The criticality is the ratio of the maximum stress (considering all load conditions) to the stress limit. It can be seen that, due to the second-order effect, the bracing size is enlarged beyond its strength requirement in order to limit excessive horizontal displacement, which leads to increased moments and which, in turn, causes undue enlargement of the vertical member sizes.

Example 2: Truss

The second example of a single-bay braced truss (Fig. 2) is used to illustrate how a nonlinear model can be used to simplify the analysis procedure while achieving a lighter design. The configuration is typical of those found in buildings or towers.

Buckling formulas are employed for the compression modeling of members (see the Appendix).[11] All vertical members belong to design group 1, the horizontal members belong to group 2, and the bracing belongs to group 3, $E = 200$ GPa, $\gamma = 78.50$ kN/m^3, and the stress limit is 200 MPa. Own weight is included. The minimum allowable cross-sectional area is 1 mm^2. In the figure the units are millimeters and Newton.

Table 2 Sizes and stress criticalities of example 2

	Linear		Nonlinear	
Group	Area	Criticality	Area	Criticality
1	133.0	1.000	136.7	1.000
2	1.000	0.273	10.95	1.000
3	53.05	1.000	$\boxed{7.127}$	1.000
Weight	98.2216		$\boxed{71.7268}$	
NCE (SQP)	20		12	
NCE (SAM)	38		18	

Table 3 Optimal design variables

Variable	w/o p_i	with p_i
a_1	7.0246	6.1861
a_2	2.1364	0.91519
a_3	2.7568	1.7772
p_1	—	−0.026807
p_2	—	−2.9666
p_3	—	−4.3438
Weight	15.96936	12.17691
NCE (SQP)	15	20
NCE (SAM)	15	N/A

In this example, cable members consisting of two two-node elements are used to model the bracing in the nonlinear structure, whereas simple two-node truss bars are used to brace the linear structure. Table 2 shows the member sizes and stress criticalities for the two models. Note the difference in weight. In the design of the linear structure the compressive member size is enlarged to prevent buckling whereas the compression member of the nonlinear example "bends" or "sags," thereby shedding all its load and transferring it to the other brace in tension. This sophistication in modeling results in a 27% overall reduction of weight. Although approximately the same result can be obtained by detecting compression failed members and artificially "removing" them from the structure, the described method is more rigorous and can be refined by implementing beam members in which the postbuckling stiffness can be retained.

Example 3: Three-Bar Prestrained Truss

To illustrate the use of prestrain variables the standard three-bar truss weight minimization problem is used. The problem definition is given in Ref. 12. The example has no buckling or slenderness constraints and self-weight is not included. The original example requires three cross-sectional design variables, one for each bar, and the structure is subjected to three load cases. The design is subjected to both stress and displacement constraints. The linear structure has an optimal weight of 20.54375, which increases to 20.55776 when a nonlinear analysis is conducted. However, because a displacement constraint of 0.005 is applied to the optimization problem, the introduction of a prestrain variable to each of the bars has no effect on the optimum design.

As a consequence, the displacement constraint was relaxed to 0.05, which yields an optimal weight of 15.96936 without prestrain variables. In this case, only the stress constraints are active. When introducing a prestrain design variable for each bar, the weight reduces to 12.17691. In this case the displacement constraint also becomes active.

The optimum design variable data are presented in Table 3.

Example 4: Simplified Antenna

The fourth example is that of a three-dimensional simplified antenna structure (see Fig. 3). The structure qualifies as a cable-stayed structure as it is unstable in its undeformed configuration. Young's modulus of the material is 200 GPa, the density is 80 kN/m^3, and the stress limit is 300 MPa. The lower bound on the cross-sectional area is 1 mm^2. Two loadcases are applied, namely, the own weight only case as well as the case of the own weight plus a vertical load of 200 N at each of the nodes 5 and 6. The vertical members have been designed according to the buckling formulas from Ref. 11 given in the Appendix. The design variable linking is shown in Table 4. All degrees of freedom at nodes 11 to 18 are fixed.

Three optimization models have been considered, namely, 1) the weight minimization using cross-sectional areas only and with no prestraining but with a displacement limit of 50 mm, 2) the minimization of the weight using both sizing and prestrain variables and with a displacement limit of 50 mm, and 3) a combined area and prestrain variable optimization of the structure to minimize its maximal displacement. In the last case a weight upper bound that is equal to the minimum weight of model 1 is specified. The results for the three design models are given in Table 5. The objective value is boxed in each case.

Table 4 Design data, example 4

Member	Joints	Group
1	2,4	1
2	1,3	
3	8,10	
4	7,9	
5	3,4	2
6	7,8	
7	4,6	3
8	6,8	
9	3,5	
10	5,7	
11	1,11	4
12	2,12	
13	9,13	
14	10,14	
15	1,19	5
16	2,20	
17	9,21	
18	10,22	
19	19,15	
20	20,16	
21	21,17	
22	22,18	

Table 5 Optimal design data of example 4

	Optimization model		
Variable	1	2	3
a_1	258.440	14.3827	211.155
a_2	155.858	10.1673	149.309
a_3	143.192	10.1727	149.309
a_4	244.766	160.233	603.921
a_5	446.410	17.6006	258.418
p_5	—	−2.14441	−2.37277
Weight	$\boxed{816.3061}$	$\boxed{131.8228}$	816.3061
Max. disp	50	50	$\boxed{4.669}$
NCE (SQP)	53	84	181
NCE (SAM)	13	N/A	N/A

Table 6 Optimal design data of example 5

	Optimization model	
Variable	1	2
a_1	20.000	34.051
a_2	85.583	59.917
a_3	20.000	20.000
a_4	124.36	115.37
a_5	114.36	102.92
p_4	—	−6.6844
Weight	$\boxed{2833.2}$	2833.2
Max. disp	5000	$\boxed{2619}$
NCE (SQP)	23	156

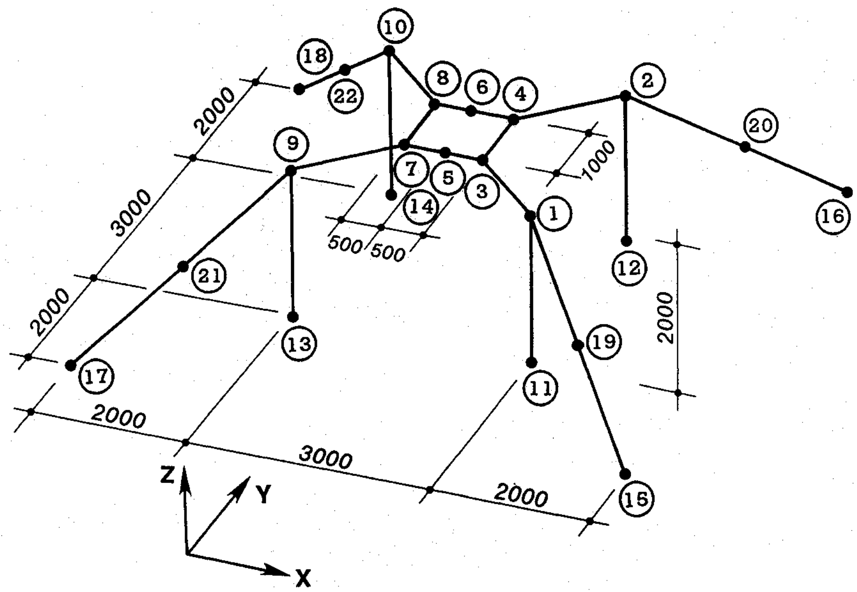

Fig. 3 Example 4: pretensioned antenna.

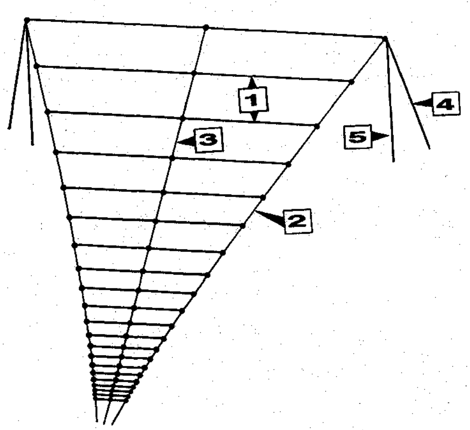

Fig. 4 Example 5: log-periodic-dipole antenna.

Example 5: Log-Periodic-Dipole Antenna

To demonstrate the optimization of a realistic structure a model of a log-periodic-dipole (LPD) antenna (see Fig. 4) is optimized. The structure has an approximate maximum height of 30 m, a length of 90 m, and a maximum width of 60 m and is subjected to self-weight only.

In this type of structure, its electromagnetic accuracy is usually of great importance and, therefore, the objective of the design is typically the minimization of the maximal displacement. Two design models were considered, namely, 1) the weight minimization utilizing sizing variables only and with a displacement upper bound of 5000 mm and 2) the minimization of the maximal displacement using both prestrain and cross-sectional variables. In this case the weight is bounded by the optimal weight of case 1. The prestrain variable is only active for design group 4. The sizing variables have a lower bound of 20 mm^2.

The numbers in the squares in Fig. 4 represent the design groups linking the members. The results for the two design models are shown in Table 6. The prestrain variables have bounds $-100 < p_4 < 0$ (millistrain).

An optimization was also conducted for case 2, but with design variables p_3 and p_4 active. The optimization yields exactly the same result as for optimization model 2, i.e., $p_3 = 0.0$ (the upper bound).

Conclusion

An integrated analysis and design method which consists of the combination of the pure Newton method and the optimization methods SQP or SAM is proposed for the design of cable-stayed truss structures, such as transmission lines or antennas.

The results demonstrate that for the purpose of design automation of skeletal structures, it becomes important to employ nonlinear analysis methods. Where secondary effects play a major role in the structural behavior, the linear analysis/optimization is not conservative and the inclusion of nonlinear geometric effects is necessary to obtain a safe optimum. In structures where compression bracing is not cost effective and load redistribution to tensile members is desirable, linear analysis is unduly conservative, and the result obtained is far from the actual optimum. Both of these phenomena are most rigorously and conveniently addressed by using nonlinear analysis and design methods.

The last three examples illustrate the effectiveness of using prestrain variables to further increase the efficiency of skeletal structures. For both of the objectives of minimum weight and minimum sag, significant reductions were obtained. Furthermore, the examples have shown that the two successive approximation methods SQP and SAM are effective in the optimal sizing and pretensioning of guyed trusses.

Appendix: Local Buckling Formula

If the slenderness ratio $\lambda \geq C_c$, then the limiting compressive stress is given by the Euler formula

$$p_c = \pi^2 E/\lambda^2 \tag{A1}$$

If the slenderness ratio $\lambda \leq C_c$, then the limiting compressive stress is given by

$$p_c = f_y\left[1 - \left(\lambda^2/2C_c^2\right)\right] \tag{A2}$$

where

$$C_c = \pi\sqrt{2E/f_y} \tag{A3}$$

where E is Young's modulus and f_y is the yield stress.

In all of the examples where buckling constraints are applied, a radius of gyration $r = \sqrt{0.25A}$ was used.

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